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MASONRY ORTHOTROPIC VAULTS IN HISTORICAL CONSTRUCTION: THE HERRING-BONE PATTERN

Barbieri A.*, Carloni C.°, Di Tommaso A.*

*University IUAV of Venice
dCa
Dorsoduro 2206
30123 Venice (Italy)

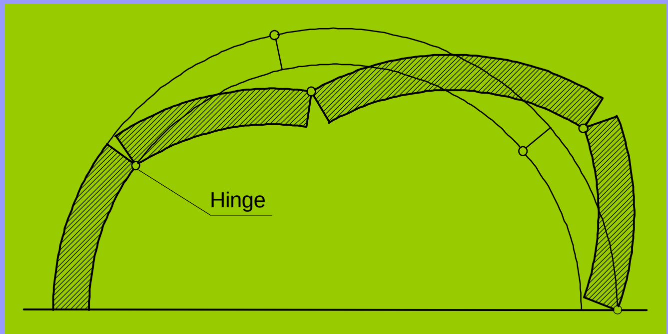
*University of Bologna
DISTART
Viale Risorgimento 2
40136 Bologna (Italy)

PRESENTATION SUMMARY

- *INTRODUCTION*
- *AIM OF RESEARCH*
- *HERRING-BONE MASONRY TECHNIQUE*
- *BENDING THEORY OF ORTHOTROPIC SHELL*
- *FE MODEL METHOD*
- *CONCLUSIONS*

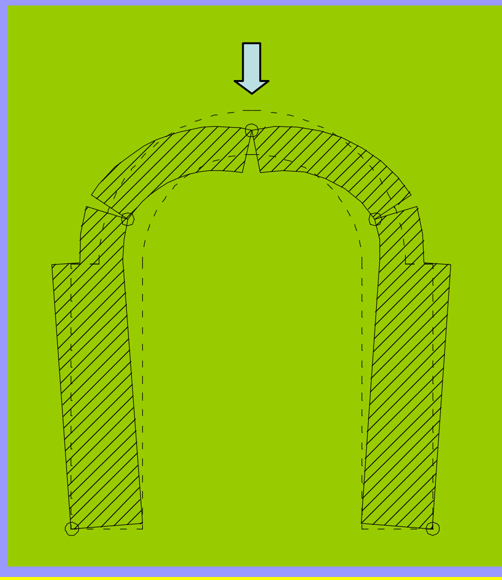
INTRODUCTION

In the masonry vaults the brick patterns could be different depending on the geometry of structural element.



The mason sensibility led to define some masonry building patterns more suitable than others because the brick pattern could contribute to reduce the support elements cost and to guarantee an increment in arch stability.

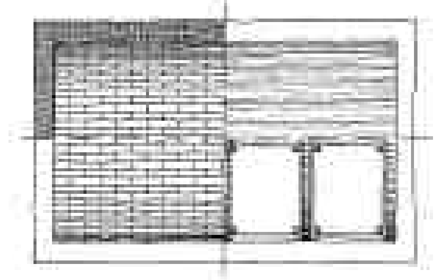
The aim of design criteria was to prevent possible kinematical modifications, because they could not made dimensional design on the base of stresses criteria (not yet formulated).



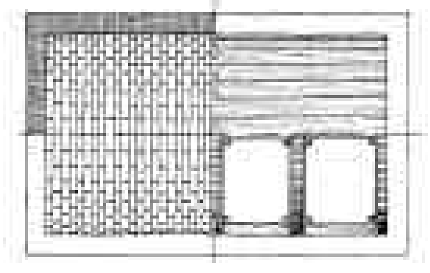
INTRODUCTION

Nevertheless, masonry vaults cover wide rooms in many historical buildings. These structural elements were built using a material which is considered as "no tesion" material.

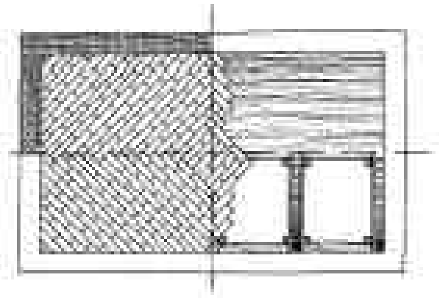
These structures need strengthening design due to loading increment and material damage. Their stability is evaluated considering collapse mechanisms and few data are available about their elastic behaviour under dead and live loadings.



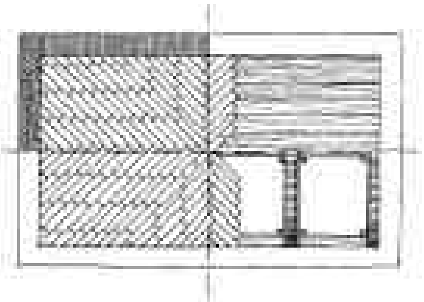
Volte a botte a filari longitudinali



Volte a botte a filari trasversali



Volta a botte a spina di pesce dritta



Volta a botte a spina di pesce inversa

Their technology is know even less, especially if the masonry has particular pattern.

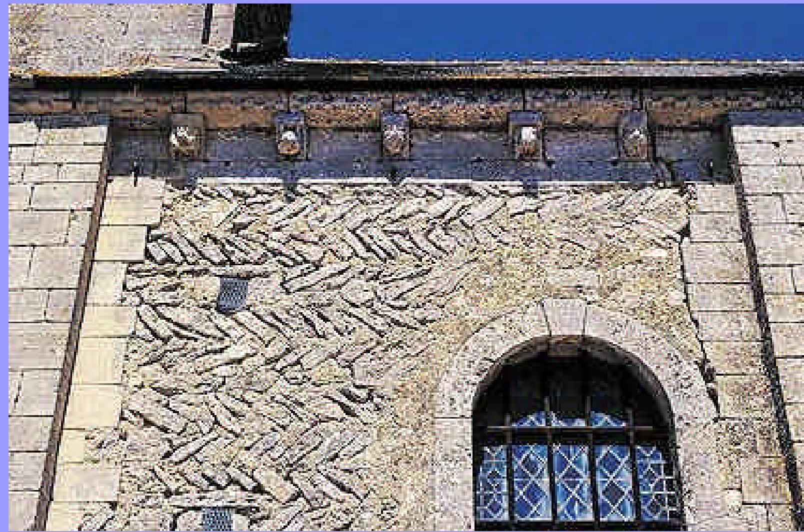
AIM OF RESEARCH

To develop an analytical model in order to investigate the structural behaviour of masonry barrel vaults built with herring-bone pattern

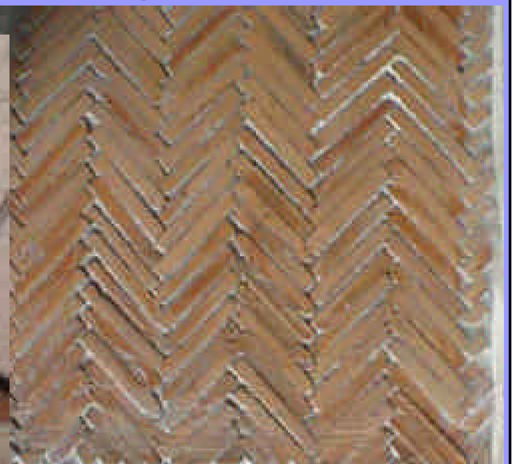
- These structural elements are analysed in the field of shell theory and linear elastic behaviour of material
- The masonry barrel vaults are modelled as bi-dimensional orthotropic elements
- The necessity to evaluate the effect of herring-bone pattern on the stresses configuration requires the introduction of flexural components in the equilibrium conditions
- The angle α of masonry configuration in herring-bone pattern is assumed as parametric element for the analysis.

HERRING - BONE MASONRY TECHNIQUE

Herringbone stonework, common in small parish churches (Vienne-en-Bessin).



Herringbone pattern for stone wall. Particular of stone disposition. Dossomaggiore Castle, sec. XIV

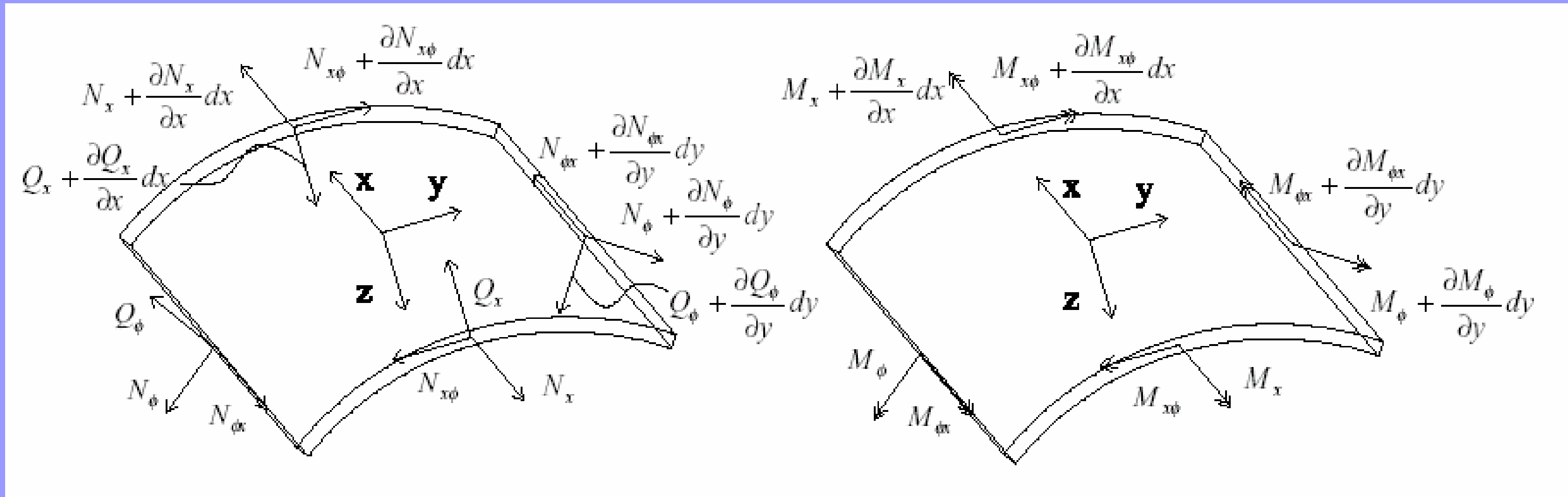


Herringbone pattern for paving.

HERRING - BONE MASONRY TECHNIQUE

Masonry barrel and crossed vaults of Tolentini monastery (Venice).

BENDING THEORY OF ORTHOTROPIC SHELL



- vault thickness is very small compared to curvature radius and length of vault;
- strains in vault are enough small so that the terms of second and over order could be neglected;
- normal stress along vault thickness is enough small compared to other stresses, so that this could be neglected;
- the normal to un-deformed vault surface is still normal in deformed configuration of vault.

BENDING THEORY OF ORTHOTROPIC SHELL

Equilibrium equations:

$$RN_{x,x} + N_{fx,f} + p_x R = 0$$

$$N_{f,f} + RN_{xf,x} + M_{xf,x} - \frac{1}{R} M_{f,f} + p_f R^2 = 0$$

$$N_f + 2M_{fx,xf} + RM_{x,xx} + \frac{1}{R} M_{f,ff} + p_r R^2 = 0$$

Strain components referring to Donnell formulation:

$$e_x = u_{x,x} \qquad e_f = \frac{u_{f,f}}{R} - \frac{u_z}{R} \qquad g_{xf} = \frac{u_{x,f}}{R} + u_{f,x}$$

$$c_x = u_{z,xx} \qquad c_f = \frac{u_{z,ff}}{R^2} \qquad c_{xf} = \frac{u_{z,xf}}{R}$$

Hooke's law:

$$s_{ij} = H_{ijlk} e_{lk}$$ *(i,j,l,k=1,2,3)*

BENDING THEORY OF ORTHOTROPIC SHELL

Equilibrium equations:

$$(H_{11}-\frac{H_{13}^{2}}{H_{33}})\cdot R\cdot u_{x,xx}+\frac{H_{66}}{R}\cdot u_{x,ff}+(H_{12}-\frac{H_{13}H_{32}}{H_{33}}+H_{66})\cdot u_{f,xf}-(H_{12}-\frac{H_{13}H_{32}}{H_{33}})\cdot u_{z,x}=0$$

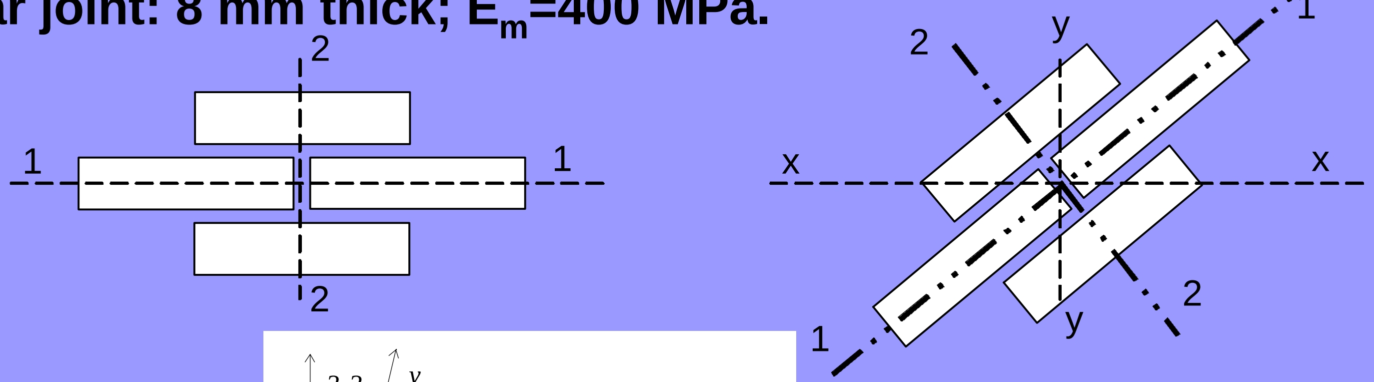
$$(H_{21}-\frac{H_{23}H_{31}}{H_{33}}+H_{66})\cdot u_{x,xf}+H_{66}\cdot R\cdot u_{f,xx}+\frac{1}{R}\cdot(H_{22}-\frac{H_{23}^{2}}{H_{33}})\cdot u_{f,ff}+\frac{t^{2}}{12\cdot R^{3}}\cdot(-H_{22}+\frac{H_{23}^{2}}{H_{33}})\cdot u_{z,fff}+$$
$$+\frac{t^{2}}{6\cdot R}\cdot(H_{66}-\frac{H_{12}}{2}+\frac{H_{23}H_{31}}{2\cdot H_{33}})\cdot u_{z,xxf}+\frac{1}{R}\cdot(-H_{22}+\frac{H_{23}^{2}}{H_{33}})\cdot u_{z,f}+p_{f}\frac{R^{2}}{t}=0$$

$$(H_{21}-\frac{H_{23}H_{31}}{H_{33}})\cdot u_{x,x}+\frac{1}{R}\cdot(H_{22}-\frac{H_{23}^{2}}{H_{33}})\cdot u_{f,f}+\frac{t^{2}\cdot R}{12}\cdot(H_{11}-\frac{H_{13}^{2}}{H_{33}})\cdot u_{z,xxxx}+\frac{t^{2}}{3\cdot R}\cdot(H_{66}+\frac{H_{12}}{2}-\frac{H_{13}H_{32}}{2\cdot H_{33}})\cdot u_{z,xxff}+$$
$$+\frac{t^{2}}{12\cdot R^{3}}\cdot(H_{22}-\frac{H_{23}^{2}}{H_{33}})\cdot u_{z,ffff}+\frac{1}{R}\cdot(-H_{22}+\frac{H_{23}^{2}}{H_{33}})\cdot u_{z}+p_{r}\frac{R^{2}}{t}=0$$

These equilibrium equations can not be solved in an analytical form, hence a numerical procedure has been proposed.

F.E. MODEL METHOD

Clay brick: 250 mm long, 60 mm high and 120 mm thick; E_b=4000 MPa
Mortar joint: 8 mm thick; E_m=400 MPa.



Mechanical characteristics of masonry

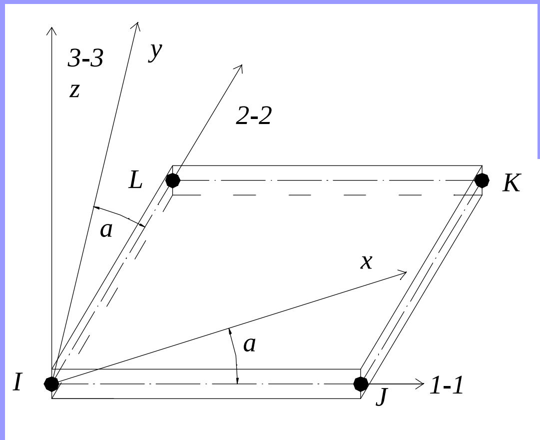
E_{11}=3290 MPa

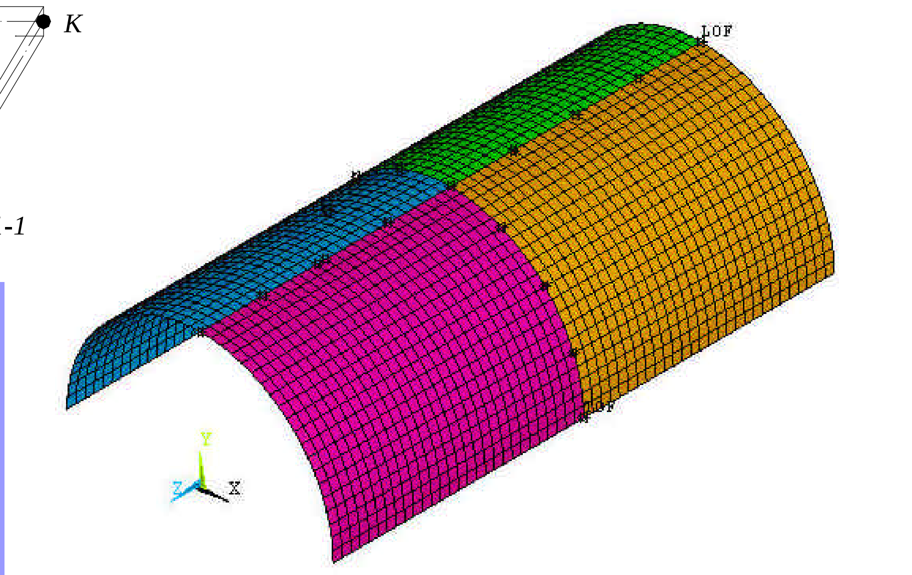
E_{22}=1775 MPa

ν_{12}= 0.111

ν_{21}= 0.206

G_{12}=1188 MPa.





F.E. MODEL METHOD

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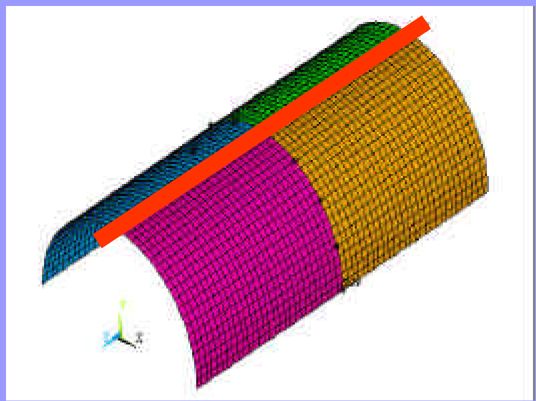
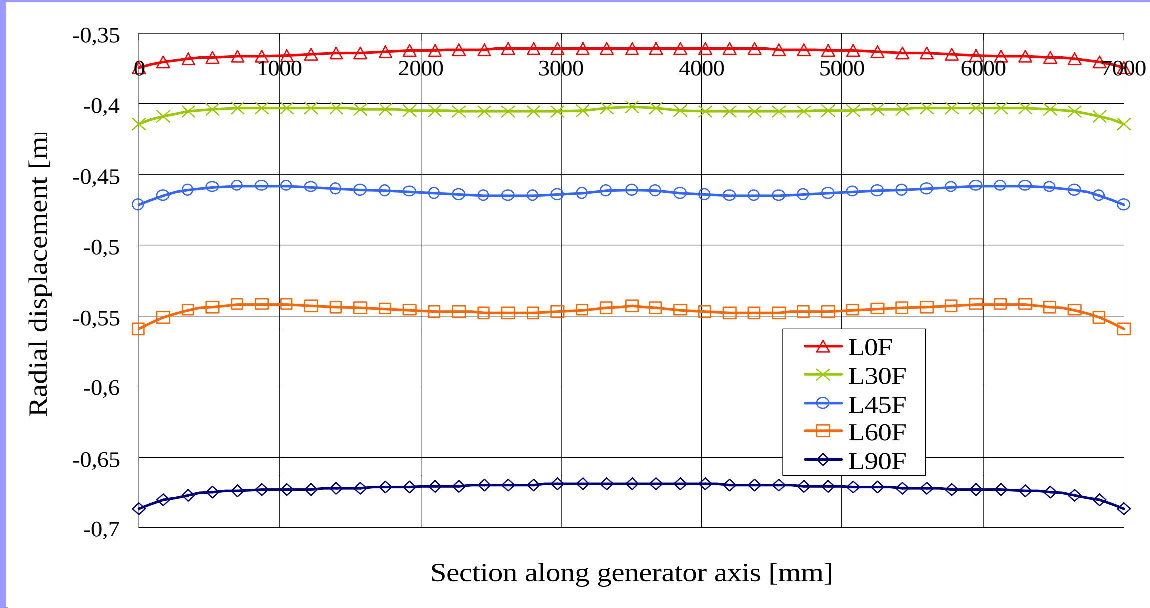
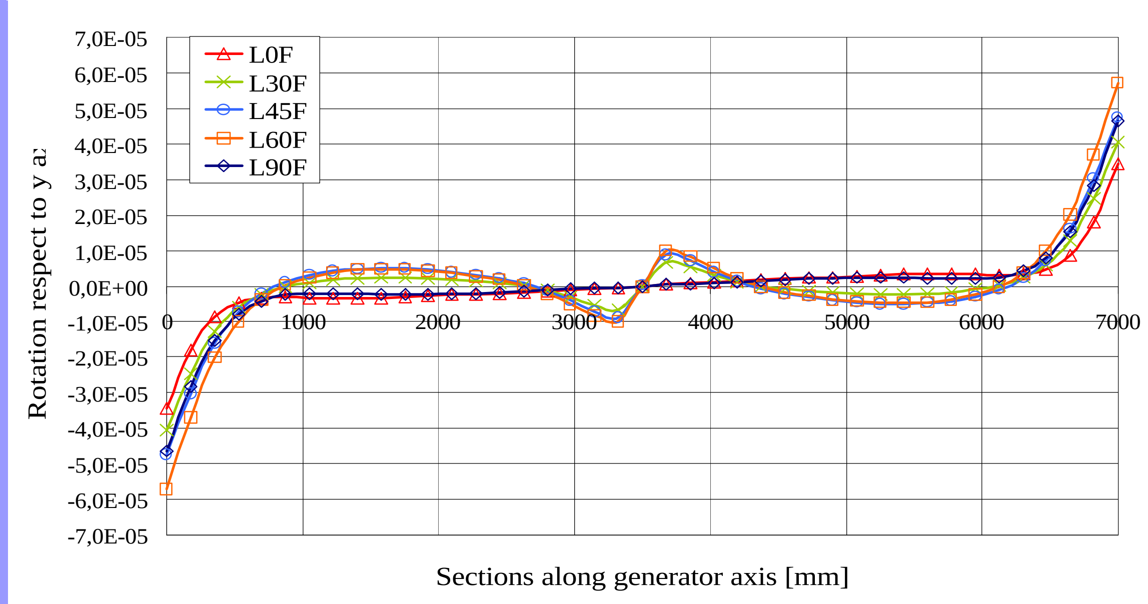
Radial displacement along generator axis at crown, for different orthotropic angle





Rotation along generator axis at crown, for different orthotropic angle



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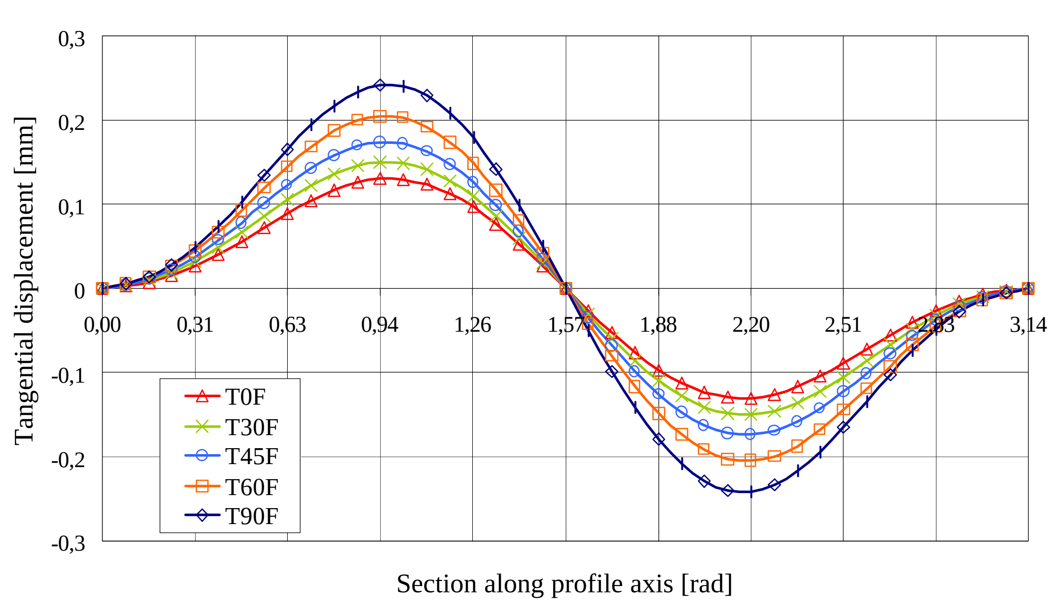
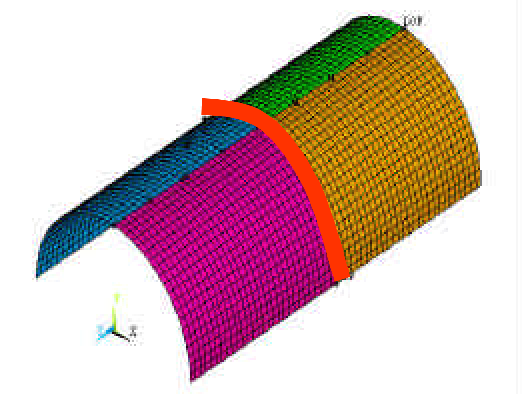
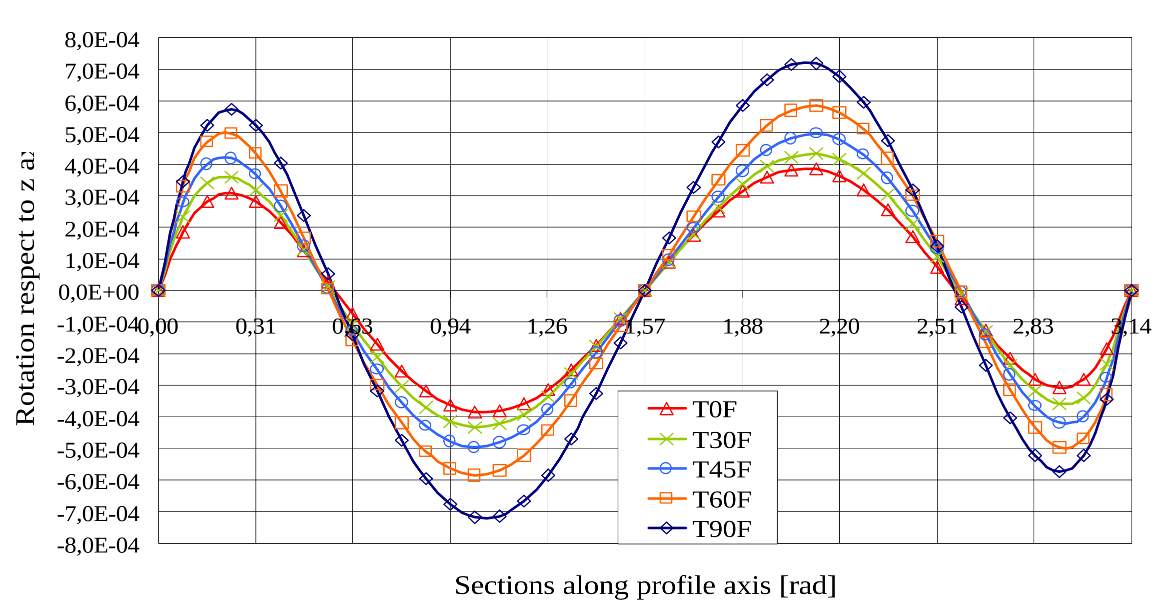
F.E. MODEL METHOD

Tangential displacement in middle arch, for different orthotropic angle





Rotation along *z* axis in middle arch, for different orthotropic angle



F.E. MODEL METHOD

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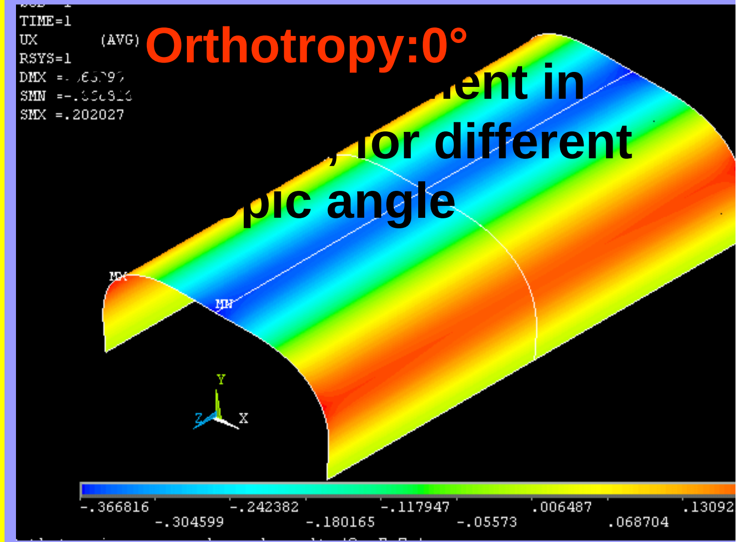

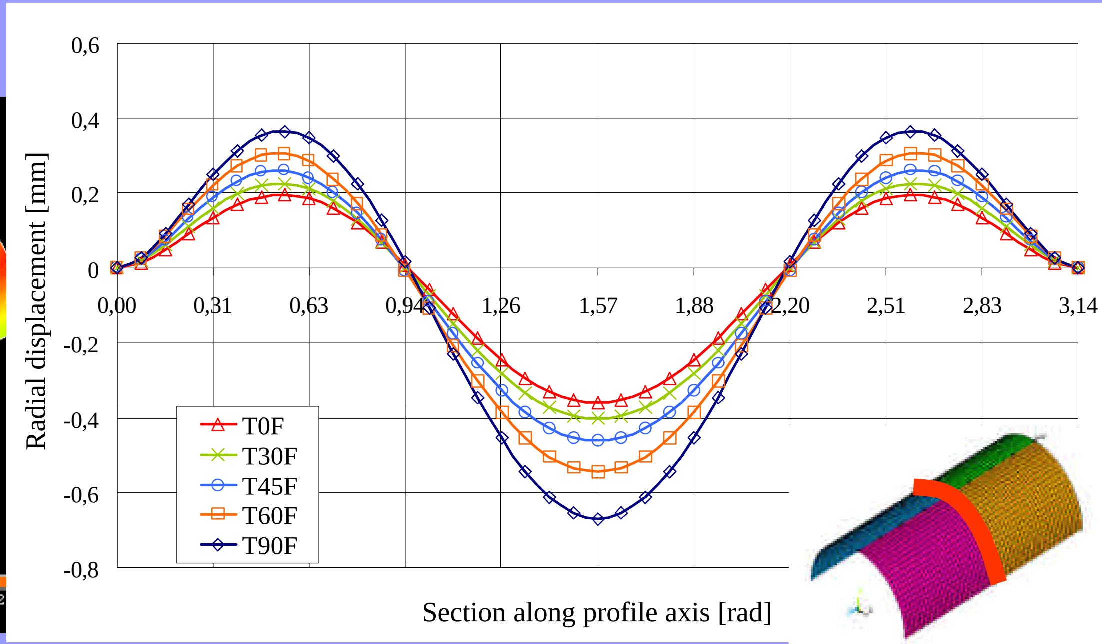

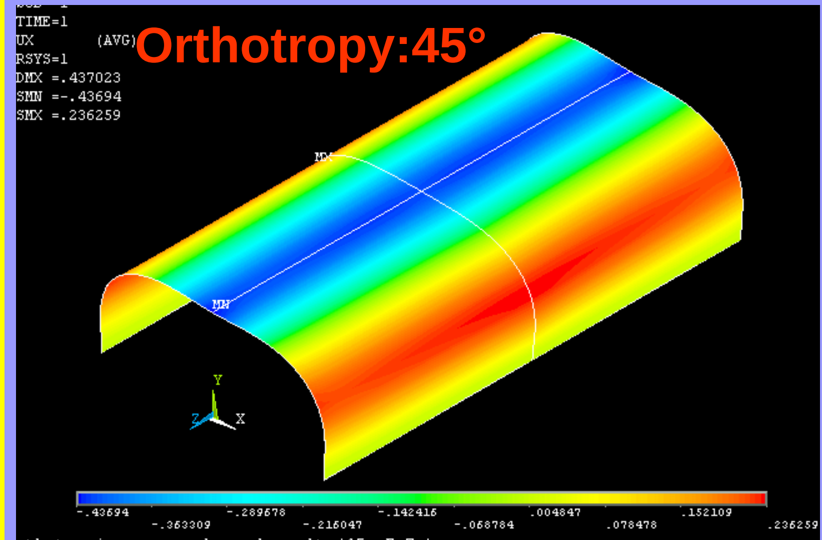

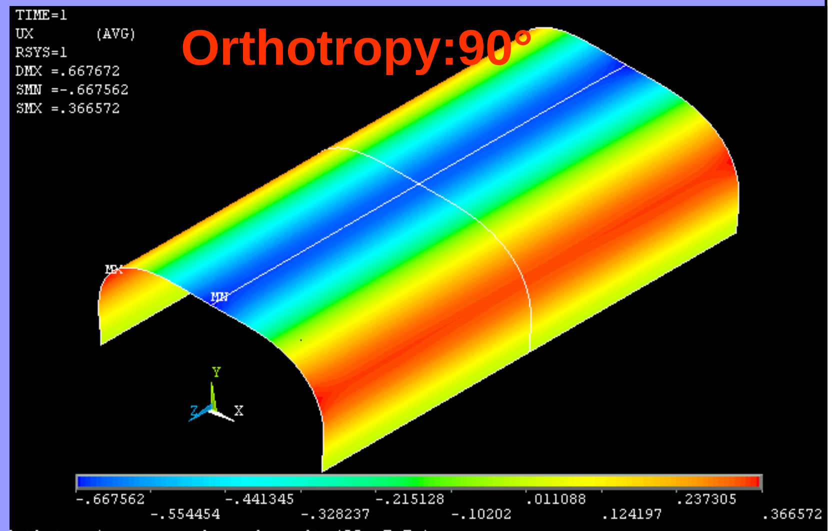

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CONCLUSIONS

✓Analytical and numerical approach in order to investigate the structural behaviour of masonry barrel vaults built with herring-bone pattern is considered.

✓The displacements components along the middle arch and the generator axis defined by ϕ=0° are represented for different values of the inclination angle of the directions of orthotropy.

✓When ϕ decreases the values of the displacements components decreases too, meaning that the barrel vault becomes more stiff.

✓Moreover the more used pattern (i.e. ϕ=90°), seems to be the one with lower stiffness.

✓Research in progress: to analyse the limit analysis of these kind of vaults, in order to study the influence of the herring bone pattern on the collapse mechanism.